Enrollment No: ______________________ **Exam Seat No:** ______________________

C.U.SHAH UNIVERSITY

Summer Examination-2017

Subject Name: Real Analysis

Subject Code: 5SC02REA1 **Branch: M.Sc. (Mathematics)**

Semester: 2 **Date: 09/05/2017** **Time: 02:00 To 05:00** **Marks: 70**

Instructions:

(1) Use of Programmable calculator and any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

SECTION – I

Q-1 Answer the Following questions: **(07)**

a) Define: σ -Algebra of sets **(02)**

b) For any countable set $A \subseteq R$ then $m^*(A) = 0$. **(02)**

c) For any $A \subseteq R$, if $m^*(A) = 0$ then A must be a measurable set. **(02)**

d) Q and Q^c are not measurable sets. – True or False? **(01)**

Q-2 Attempt all questions **(14)**

a) Let m be the set of all measurable subsets of R then prove that m is an algebra of R. **(07)**

b) Prove that outer measure of an interval is its length. **(07)**

OR

Q-2 Attempt all questions **(14)**

a) Let $\mathcal{A}$ be an algebra on X and $\{A_i\} \in \mathcal{A}$ then there exist $\{B_i\} \in \mathcal{A}$ such that **(05)**

i) $\bigcup_{i=1}^{n} A_i = \bigcup_{i=1}^{n} B_i$ and ii) $B_i \cap B_j = \phi$, for $i \neq j$.

b) Give an example of non-measurable set and explain. **(09)**

Q-3 Attempt all questions **(14)**

a) If $E_1, E_2, ..., E_n$ be a finite sequence of measurable sets and they are mutually disjoint then for any $A \subseteq R$, $m^*\left(A \cap \left(\bigcup_{i=1}^{n} E_i\right)\right) = \sum_{i=1}^{n} m^*(A \cap E_i)$ **(05)**

b) Let ϕ and ψ are simple functions on E which are vanish outside of a set of finite measure then prove that $\int a\phi + b\psi = a\int \phi + b\int \psi$. **(05)**

c) Give an example of an algebra which is not an σ-Algebra of X and explain. **(04)**

OR

Q-3 Attempt all questions **(14)**

a) Give an example of measurable map which is not a Riemann integrable map and explain. **(05)**

b) Prove that every borel set is a measurable set. **(05)**

c) Let f, g be two measurable function on *E*, where $E \in$ m then for any $f + c, cf$ & $f + g$ are also measurable function on *E*. **(04)**

SECTION – II

Q-4 Answer the Following questions: **(07)**

a) State Fatou's lemma. **(02)**

b) Define: Lebesgue integral. **(02)**

c) Define: Positive part and negative part of function. **(02)**

d) Define: $BV[a,b]$. **(01)**

Q-5 Attempt all questions **(14)**

a) State and prove Bounded convergence theorem. **(07)**

b) Suppose $f, g \in BV[a,b]$ then prove that the following: **(07)**

i) $fg \in BV[a,b]$ and ii) $\frac{f}{g} \in BV[a,b], where\, g \neq 0$.

OR

Q-5 Attempt all questions **(14)**

a) State and prove monotone convergence theorem. **(07)**

b) State and prove Lebesgue dominated convergence theorem. **(07)**

Q-6 Attempt all questions **(14)**

a) F is absolutely continuous function on $[a,b]$ iff F is indefinite integral. **(07)**

b) State and prove Beppo-Levi's theorem. **(04)**

c) State J E Littlewood's three principles. **(03)**

OR

Q-6 Attempt all Questions **(14)**

a) Let f be a bounded measurable function on $[a,b]$ and $F(x)=\int_a^x f(t)\,dt+F(a)$ **(07)**

then $F'(x)=f(x)$ almost everywhere on $[a,b]$.

b) If f is integrable over E then $|f|$ is integrable over E and $\left|\int_E f\right| \le \int_E |f|$. **(04)**

c) If $A, B \subseteq R$ be such that $m^*(A)=0$ then prove that $m^*(A\cup B)=m^*(B)$. **(03)**